Grade Level/Course: Grades 6-7

Lesson/Unit Plan Name: Part 2 - Multiple Representations of Ratios: from concrete to operational.

Rationale/Lesson Abstract: How comparing and contrasting multiple representations shift student's thinking from concrete to operational. Using four models to create and understand the relationship of ratios in four different models. Then use the models comparing and contrasting them to understand how each can be used and what each of them offer in understanding ratios.

Timeframe: 2 days to multiple days

Common Core Standard(s):

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.

A duck flew at 18 miles per hour for 3 hours, then at 15 miles an hour for 2 hours. How far did the duck fly in all?

A. 69 miles

B. 75 miles

C. 84 miles

D. 81 miles

6.RP.3, 7.RP.1, 7.RP.2c

Solve

$$\frac{5}{150} = \frac{x}{1025}$$

7.RP.2

Describe and correct the error in solving the proportion.

$$\frac{m}{10} = \frac{50}{20}$$

$$10 \times \frac{m}{10} = 20 \times \frac{50}{20}$$

$$m = 50$$

7.RP.2a

Write a proportion:

You can buy 6 bottles of water for $5.00. You can buy X bottles of water for $12.50.

What are some common errors made in writing a proportion?

6 RP.3, 7.RP.2b

A duck flew at 18 miles per hour for 3 hours, then at 15 miles an hour for 2 hours. How far did the duck fly in all?

$$18\times 3 + 15 \cdot 2 = 84 \text{ miles}$$

A. 69 miles

B. 75 miles

(C.) 84 miles

D. 81 miles

6.RP.3, 7.RP.1, 7.RP.2c

Solve

$$\frac{5}{150} = \frac{x}{1025}$$

$$\frac{5}{2\cdot 3\cdot 5\cdot 5} = \frac{5}{2\cdot 3\cdot 5\cdot 5\cdot 7}$$

$$\frac{5}{2\cdot 3\cdot 5\cdot 5} = \frac{5}{2\cdot 3\cdot 5\cdot 5\cdot 7}$$

$$x = 35$$

7.RP.2d

Describe and correct the error in solving the proportion.

$$\frac{m}{10} = \frac{50}{20}$$

$$10\cdot\frac{m}{20} = 20\cdot\frac{50}{20}$$

$$m = 50$$

Both sides need to be multiplied by the same number, either 10 or 20.

7.RP.2a

Other 6.RP.3, 7.RP.2b

Write a proportion:
You can buy 6 bottles of water for \$5.00. You can buy x bottles of water for \$12.50.

$$\frac{6}{5.00} = \frac{x}{12.50}$$

$$\frac{6}{x} = \frac{5.00}{12.50}$$

What are some common errors made in writing a proportion? Placing the terms incorrectly.

6 RP.3. 7.RP.2b

Part 2, Lesson 1 - Multiple Representations of Ratios

Notes to Teacher: Students should already have had short lessons on ratio language, how to write ratios, and how to convert ratios to unit ratios and percent so they can apply that knowledge to these lessons.

These ratio lessons will show how questions and analysis of multiple methods of solving a problem deepen students' problem solving knowledge by forging connections between the methods. This also increases their flexibility with number by changing their thinking from additive to operational through the application of the multiplicative relationship of ratios and proportions. By exploring the concepts of ratio and proportion through multiple methods they take another step in becoming more confident and proficient in manipulating numbers and seeing patterns. This, in turn, increases their willingness to experiment with numbers.

Students will be able to:

<u>Part 1, Lessons 1-4</u>

- Use and manipulate concrete objects and visual tools that are instructional in formulating and testing their thinking and understanding of ratio and proportion as multiplicative rather than additive.
- Increase their ability to think logically about the abstract concepts of ratio, rate and proportion.

<u>Part 2, Lesson 1</u>

- Recognize that using a proportion in isolation to solve a rate problem limits their knowledge to the answer of that specific problem. (This experience can be expanded by solving the proportion in 2 or 3 different ways, but does little to expand their knowledge of rate.)
- Use a table to reveal the answers of the unit rate, and realize that those rates that are usually whole number multiple terms in between the proportion given and the proportion sought. (Ex. The ratio 3 to 5. Thinking: If I add another 3 to 3 it makes 6 and if I add another 5 to 5 it makes 10 therefore the next equivalent ratio is 6 to 10.)
- Use a graph to visually understand ratio as a linear model with answers as numerous as the points on a line.
- Use an algebraic equation to express the generalization of the pattern and to solve for any value.

Part 2, Lesson 1 – Multiple Representations of Ratios

Materials

- Your favorite way to impart group lessons. (white board, document camera),
- chart paper,
- markers,
- the lesson worksheet included in this lesson.

Lesson 1: The lesson begins with a word problem involving rate set into a Frayer model of multiple methods. After completing each method (from right to left in order) there is a line of guided questions for you to help students explore how the different methods support and evolve in their ability to inform understanding.

Pass out copies of the rate word problem to your students. Read the problem aloud. Have students talk at their table about how to set the problem up as a proportion. Have them share out how they set up the proportion equation and why.

$$\frac{2}{30} = \frac{x}{120}$$

$$2 \cdot 120 = 30 \cdot x$$

$$40 = 30x$$

$$\frac{\mathbf{240}}{\mathbf{30}} = \frac{\mathbf{30x}}{\mathbf{30}}$$

PROPORTION Students should set up the problem in the proportion section.

Have them say the problem in the first box labeled proportion as “2 is to 30 as X is to 120.” Proceed with doing a “Think, Pair, Share” on the first problem. *“How will we set up the problem as a proportion?”*

While they are doing this circulate and decide who you will have share their answers. Don’t forget there is learning in misconceptions as well as correct answers.

TABLE

minutes	**30**	**60**	**90**	**120**	**150**
miles	**2**	**4**	**6**	**8**	**10**

Have students determine the labeling of their table and insert the numbers in the appropriate places in the table.

Ask *“What do you think should go in the next box?”*

Then ask them to *“Talk to your group and decide what the next term should be and why you believe that your answer is correct. Make sure everyone at your table understands and can justify the answer.”* Have several students share out the terms to be put within the table and then have other students in the group share the group’s justification for the placement of those numbers.

Part 2, Lesson 1 – Multiple Representations of Ratios

Comparing and Contrasting the Two Models

Put up two sheets of chart paper side by side and divide each sheet in ½ lengthwise so you will have a side by side comparison of all 4 models in the end. Use chart paper to record student answers as they compare and contrast the first **2** models.

Questions:

How are the models alike?

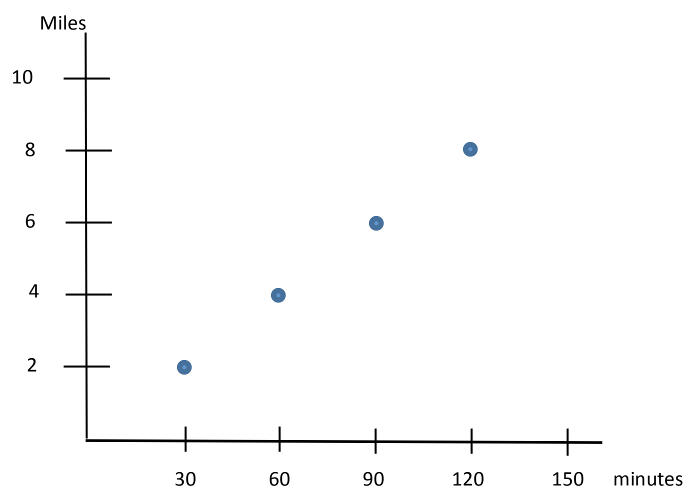
How are they different?

Does one reveal more about the problem than the other?

If so, how does it inform your understanding of the problem?

Which one would you use if there were more questions about how far Lin could run?

Do either of these have limitations on what they can tell you?



GRAPH

Ask students:

What information do we need to record on the graph?

How can we do it?

What is the incremental change needed for each piece of information?

Does it make any difference which axis we use for either information?

Have them create their graph and record their information.

Part 2, Lesson 1 – Multiple Representations of Ratios

Now use the 2nd piece of chart paper to record student answers about the graph as they compare and contrast the ***3*** *models that have now been presented. Record under the appropriate method any comments made about that method.*

What information does your graph show?

How does it compare to the proportion method?

How does it compare to the table method?

Does it tell you more or less than the others or are they all comparable?

Is it more like one than the other?

EQUATION $\boldsymbol{y = \frac{1}{15}x}$

Ask students:

What information do we need to know to put an equation together?

Which of these methods were helpful in discovering what the rule is for this problem?

How can we use the information to figure out an equation so that, no matter how far she runs, we can always use this equation, plug in the values, and find the correct answer?

Now include on the 2nd piece of chart paper student answers about the equation as they compare and contrast the ***4*** *models that have now been presented. Record under the appropriate method any comments made about that method*

When finished with all 4 methods ask these questions:

Having done all four methods, did any of them increase your understanding more than the others?

Was it important to your understanding to do all of them? Are there any you would have left out? Why?

Is there a problem that limits what you learn from it? Does it still have value?

Do you have a favorite method from this exercise? If so, which one and why?

Part 2, Lesson 1 – Multiple Representations of Ratios

<u>You try</u>

A recipe for potato salad calls for 30 potatoes to make 12 quarts of potato salad. How many potatoes are required to make 4 quarts of potato salad?

<u>Homework</u>

When 2 full moons occur in the same month the second full moon is called a blue moon. On average, two blue moons happen every five years. Find how many blue moons are likely to occur in the next thirty-five years.

and/or

Anthony drove two hundred forty miles in four hours. If he drove six hundred and twenty-four miles at the same rate of speed the next day, how long did he drive?

Part 2, Lesson 1 – Multiple Representations of Ratios

Worksheet

Proportion

— = —

Table

Lin can run 2 miles in 30 minutes.

How far can she run in 120 minutes?

Graph

Equation

Part 2, Lesson 1 – Multiple Representations of Ratios

Worksheet (Solutions)

Proportion

$$\frac{2}{30} = \frac{x}{120}$$

$$2 \text{ x } 120 = 30x$$

$$240 = 30x$$

$$\frac{240}{30} = \frac{30x}{30}$$

$$8 = x$$

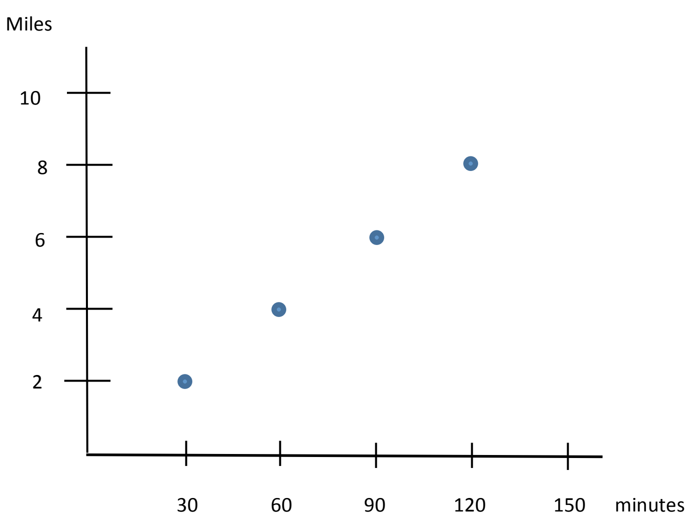
Table

30	60	90	120	150
2	4	6	8	10

Lin can run 2 miles in 30 minutes.

How far can she run in 120 minutes?

Graph



Equation

Let *x* be minutes and *y* be miles

$$y = \frac{1}{15}x$$

Part 2, Lesson 1 – Multiple Representations of Ratios

You Try

Proportion

Table

A recipe for potato salad calls for 30 potatoes to make 12 quarts of potato salad. How many potatoes are required to make 4 quarts of potato salad?

Graph

Equation

Part 2, Lesson 1 – Multiple Representations of Ratios

You Try (Solutions)

Proportion

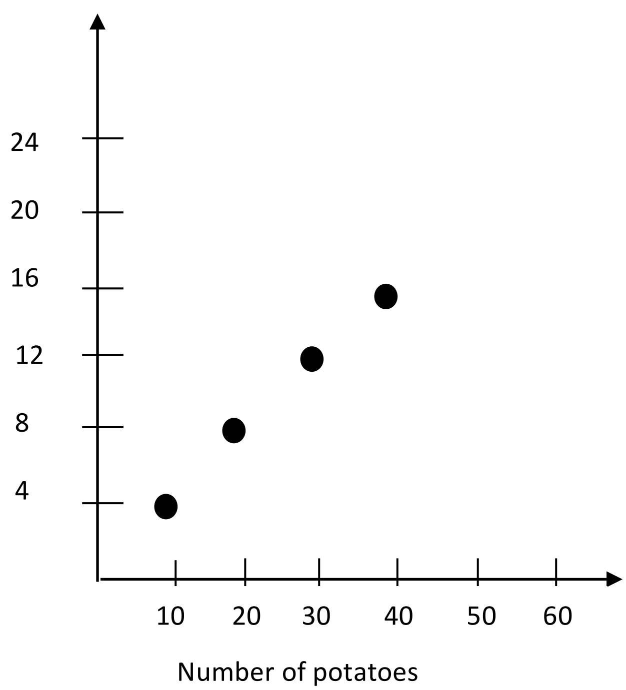
$$\frac{12}{30} = \frac{4}{x}$$

Table

Potatoes	*2.5*	5	7.5	10	20	30
Quarts of potato salad	1	2	3	4	8	12

A recipe for potato salad calls for 30 potatoes to make 12 quarts of potato salad. How many potatoes are required to make 4 quarts of potato salad?

Graph



Equation

Let x be # of potatoes and y be quarts of potato salad.

$$y = \frac{2}{5}x$$